One-Bit Radar Processing for Moving Target Detection

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***Abstract*—Target parameter estimation is a key problem in radar signal processing. In this paper, we put forward a novel approach to estimate the moving target parameters from the associated one-bit sampled data with time-varying thresholds, where one-bit samples are produced by comparing the signal of interest with a threshold level at a very high rate. We show that as the length of the transmit sequence increases, the performance of the proposed algorithm enhances to the point that it reaches that of the case of having the data with infinite precision—thus demonstrating great potential for one-bit techniques in radar signal processing applications.**

I. Introduction

Quantization is typically the first step in modern signal processing applications. An analog signal is sampled at a high rate by an analog-to-digital converter (ADC) and its magnitude is mapped to the closest pre-defined level. In many modern applications, such as spectral sensing [1], cognitive radio [2], radio astronomy [3], and automotive short-range radars [4], the signal of interest has a wide bandwidth and requires the sampling resolution to be very high. However, the cost of ADCs rise exponentially with sampling resolution and the number of quantization levels [5], which makes them impractical for many modern applications. Thus, in order to overcome these problems, the number of quantization bits should be decreased. In the most extreme case, the number of quantization bits is reduced to just one. In other words, the ADC is replaced with an inexpensive one-bit comparator that can efficiently sample at very high rates [5]. The need for enhanced resolution is thus addressed by relatively high sampling rates.

The problem of parameter estimation using one-bit sampling has been previously studied from different perspectives, including statistical signal processing [6], [7], and modern compressive sensing [8]–[10]. Until recently, the signal was compared to a fixed threshold, typically zero, which translates to loss of signal amplitude. Conversely, many recent studies focus on comparing the signal of interest with a time-varying threshold that enables the recovery of the signal in its entirety [10]–[13].

In this paper, we study the problem of estimating the parameters of a moving target radar using only the one-bit sampled data of the signal backscattered from it. In Section II, we introduce the system model and in Section III, we develop a method based on arcsine law and Bussgang theorem for the recovery of the target parameters. Then, in Section IV, we develop an algorithm to accomplish task of target parameter estimation by using designed time-varying thresholds and *a priori* knowledge of some statistics of the interference. The algorithm forms and solves an optimization problem with linear constraints that can efficiently be solved cyclically to recover the target parameters as well as the received signal. Finally, numerical results are provided in Section V to verify the efficiency of the proposed algorithm.

Notation: Bold lowercase and bold uppercase letters are used to denote vectors and matrices, respectively. $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and Hermitian transpose of a matrix argument. Additionally, $(\cdot)^*$ denotes the complex conjugate of a complex matrix, vector, or number. $\|\cdot\|$ denotes the l_2 norm of a vector while $\|\cdot\|_F$ denotes the Frobenius norm of a matrix. $\Re(\cdot)$ and $\Im(\cdot)$ are the real and imaginary parts of a complex number, respectively. $\text{sgn}(\cdot)$ is the element-wise sign operator with an output of $+1$ for nonnegative numbers and -1 otherwise. $\mathbb{E}\{\cdot\}$ stands for the expectation operator. $\mathcal{N}(\cdot)$ is the autocorrelation function normalization operator. Finally, The symbol $\odot$ represents the Hadamard product of matrices.

II. System Model

Let $\mathbf{s} = \begin{bmatrix} s_1 & s_2 & \dots & s_N \end{bmatrix}^T$ be the complex-valued transmit sequence that will be modulated as a train of pulses in time [14]. Note that the transmitted or received signal can be expressed in discrete time or digital form and all forms of processing can too be applied in digital domain. Thus, in this paper, we use the discrete model as well to represent the data; for further information see [15], [16].

The received baseband signal $\mathbf{y} \in \mathbb{C}^N$ of a moving target corresponding to the range-azimuth cell of interest can be modeled as follows [16]–[20]:

$$\mathbf{y} = \alpha_0[\mathbf{s} \odot \mathbf{p}(\nu)] + \mathbf{c} + \mathbf{n} \tag{1}$$

where $\alpha_0 \in \mathbb{C}$ is the backscattering coefficient of the target. Also, $\mathbf{p}(\nu) = \begin{bmatrix} e^{j2\pi(0)\nu}, & e^{j2\pi(1)\nu}, & \dots, & e^{j2\pi(N-1)\nu} \end{bmatrix}^T$ is the temporal sampling vector with $\nu \in [-.5, .5)$ being the normalized Doppler shift of the target. The terms $\mathbf{c}$ and $\mathbf{n}$ refer to the signal-dependent and signal-independent interferences, respectively.

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The clutter vector $\mathbf{c}$ is built by aggregation of zero-mean unwanted echoes of the transmitted signal at different range-azimuth bins [19], which can be formulated as

$$\mathbf{c} = \sum_{k=0}^{N_c-1} \sum_{l=0}^{L-1} \alpha_{(k,l)} \mathbf{J}_k \left[\mathbf{s} \odot \mathbf{p}(\nu_{d_{(k,l)}})\right] \tag{2}$$

with $N_c \leq N$ being the number of range-rings, L being the number of different azimuth sectors, and $\alpha_{(k,l)}$ and $\nu_{d_{(k,l)}}$ denoting the uncorrelated scattering coefficient and normailized Doppler shift of the (k,l) range-azimuth bin. It is assumed that the clutter patches in each range-azimuth bin have uniform Doppler shift in the interval $\Omega_c = \left(\bar{\nu}_{d_{(k,l)}} - \frac{\epsilon_{d_{(k,l)}}}{2}, \bar{\nu}_{d_{(k,l)}} + \frac{\epsilon_{d_{(k,l)}}}{2}\right)$ [20]. The matrices $\{\mathbf{J}_k\}$ are shift matrices defined as

$$\mathbf{J}_k = \mathbf{J}_{-k}^H = \begin{bmatrix} 0 & \dots & 0 & 1 & \dots & 0 \\ \vdots & & & & \ddots & \\ & & & & & 1 \\ \underbrace{0 \ \dots \ 0}_{k} & \dots & & & & \end{bmatrix}. \tag{3}$$

The covariance matrix of the clutter, $\mathbf{c}$, can be formulated as [20]

$$\boldsymbol{\Sigma}_{\mathbf{c}} = \sum_{k=0}^{N_c-1} \sum_{l=0}^{L-1} \sigma_{(k,l)}^2 \mathbf{J}_k \boldsymbol{\Phi}\left(\mathbf{s}, (k,l)\right) \mathbf{J}_k^T \tag{4}$$

where $\sigma_{(k,l)}^2$ is the average scattering power of the (k,l) range-azimuth bin. Moreover,

$$\boldsymbol{\Phi}\left(s, (k,l)\right) = \mathbf{Diag}(\mathbf{s}) \mathbf{C}_\nu(k,l) \mathbf{Diag}(\mathbf{s})^H$$

where $\mathbf{Diag}(.)$ is a diagonal matrix with diagonal entries equal to those of its vector argument and $\mathbf{C}_\nu(k,l)$ is the propagation covariance matrix of the (k,l) bin [20] that can be written as

$$\mathbf{C}_\nu(k,l) = \begin{cases} 1 & k = l \\ e^{j(k-l)\bar{\nu}_{d_{(k,l)}}} \frac{\sin\left(\frac{k-l}{2}\epsilon_{d_{(k,l)}}\right)}{\frac{k-l}{2}\epsilon_{d_{(k,l)}}} & k \neq l \end{cases}. \tag{5}$$

The covariance matrix of the zero-mean signal-independent vector $\mathbf{n}$ is denoted by

$$\boldsymbol{\Gamma} = \mathbb{E}\{\mathbf{n}\mathbf{n}^H\}. \tag{6}$$

Therefore, the covariance matrix of the interference terms in the received signal $\mathbf{y}$ can be written as

$$\mathbf{R} = \mathrm{Cov}(\mathbf{c} + \mathbf{n}) = \boldsymbol{\Sigma}_{\mathbf{c}} + \boldsymbol{\Gamma}. \tag{7}$$

By applying one-bit sampling to the received signal, i.e. comparing the received signal with pre-defined threshold levels, we obtain the one-bit sampled data given as

$$\begin{aligned} \boldsymbol{\gamma}_r &= \mathrm{sgn}\left(\Re\{\alpha_0(\mathbf{s} \odot \mathbf{p}(\nu)) + \mathbf{c} + \mathbf{n} - \boldsymbol{\lambda}\}\right), \\ \boldsymbol{\gamma}_i &= \mathrm{sgn}\left(\Im\{\alpha_0(\mathbf{s} \odot \mathbf{p}(\nu)) + \mathbf{c} + \mathbf{n} - \boldsymbol{\lambda}\}\right), \\ \boldsymbol{\gamma} &= \tfrac{1}{\sqrt{2}}(\boldsymbol{\gamma}_r + j\boldsymbol{\gamma}_i), \end{aligned} \tag{8}$$

where $\boldsymbol{\lambda} \in \mathbb{C}^N$ is the time-varying threshold vector—whose design is discussed in Section IV.

If the received signal $\mathbf{y}$ is available and the normalized Doppler shift ν is known, we can take advantage of the signal model in (1) to find an estimate of the backscattering coefficient α_0. In fact, one can use a matched filter (MF) to estimate α_0. However, in order to make the estimate more accurate and more robust to signal-independent noise, such as jammers that operate in certain frequency bands, one can utilize an instrumental variable (IV) filter, also called mismatched filter (MMF), instead of a matched filter. By using an IV filter, the estimate of the backscattering coefficient α_0 will be [24]

$$\hat{\alpha}_0 = \frac{\mathbf{w}^H \mathbf{y}}{\mathbf{w}^H(\mathbf{s} \odot \mathbf{p}(\nu))} \tag{9}$$

with $\mathbf{w} \in \mathbb{C}^N$ being the receive (IV) filter. Thus, the mean-square-error (MSE) of the estimate can be written as

$$\begin{aligned} \mathrm{MSE}(\hat{\alpha}_0) &= \mathbb{E}\left\{\left|\frac{\mathbf{w}^H \mathbf{y}}{\mathbf{w}^H(\mathbf{s} \odot \mathbf{p}(\nu))} - \alpha_0\right|^2\right\} \\ &= \frac{\mathbf{w}^H \mathbf{R} \mathbf{w}}{\left|\mathbf{w}^H(\mathbf{s} \odot \mathbf{p}(\nu))\right|^2}. \end{aligned} \tag{10}$$

It can easily be verified that the numerator of the MSE is the power of interferences and its denominator is the power of the signal at the receiver end; hence, minimizing the MSE is equivalent to maximizing the signal-to-clutter-and-interference-ratio (SCIR).

In the following, we first discuss a Bussgang-theorem and arcsine-law based algorithm to estimate the radar parameters in Section III. Then, in Section IV, we propose an algorithm to jointly estimate the radar parameters, the scattering coefficient α_0 and normalized Doppler shift ν, as well as the received signal $\mathbf{y}$ from its one-bit sampled version $\boldsymbol{\gamma}$.

III. Bussgang-theorem-aided Estimation

In this section, we put forward an algorithm using the Bussgang theorem and arcsine law to estimate the radar parameters.

Let $Y(t)$ be a real-valued, scalar, and stationary Gaussian process. If $Y(t)$ is one-bit sampled, we get the process $Z(t) = \mathrm{sgn}(Y(t))$. The autocorrelation function of the process $Z(t)$, which is denoted by $R_Z(\tau)$, is given by

$$R_Z(\tau) = \mathbb{E}\{Z(t+\tau)Z(t)\} = \frac{2}{\pi}\sin^{-1}\left(\bar{R}_Y(\tau)\right) \tag{11}$$

with $\bar{R}_Y(\tau) = R_Y(\tau)/R_Y(0)$ being the normalized autocorrelation function of $Y(t)$ [21].

The Bussgang theorem [22] shows that $R_{YZ}(\tau)$, the cross-correlation function of the two processes $Y(t)$ and $Z(t)$, is proportional to $R_Y(\tau)$, the autocorrelation function of the process $Y(t)$ which has been one-bit sampled. In other words, the equality $R_{ZY}(\tau) = \mu R_Y(\tau)$ holds with μ being the proportion factor that depends on the power of the process $Y(t)$.

As for the complex-valued case, let $\mathbf{y}$ be a complex-valued vector and denote its one-bit sampled version with $\boldsymbol{\gamma} =$

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$\frac{1}{\sqrt{2}}(\text{sgn}(\Re(\mathbf{y})) + j\text{sgn}(\Im(\mathbf{y})))$. Let $\bar{\mathbf{R}}_{\mathbf{y}}$ denote the normalized autocorrelation function of the vector:

$$\bar{\mathbf{R}}_{\mathbf{y}} = \mathcal{N}(\mathbf{R}_{\mathbf{y}}) \triangleq \mathbf{D}^{-1/2}\mathbf{R}_{\mathbf{y}}\mathbf{D}^{-1/2} \tag{12}$$

with $\mathbf{D} = \mathbf{R}_{\mathbf{y}} \odot \mathbf{I}$, i.e. $\mathbf{D}$ is a diagonal matrix whose diagonal elements are identical to those of $\mathbf{R}_{\mathbf{y}}$. According to the arcsine law [23], the following equality holds for the autocorrelation functions as well as covariance matrices:

$$\bar{\mathbf{R}}_{\mathbf{y}} = \sin\left(\frac{\pi}{2}\mathbf{R}_{\boldsymbol{\gamma}}\right). \tag{13}$$

Therefore, the above relation can be used for our purpose of estimating the radar parameters from the associated one-bit sampled data. In order to do so, we calculate the covariance matrix of the data after thresholding but before undergoing one-bit sampling, (i.e. $\mathbf{y}$ - $\boldsymbol{\lambda}$):

$$\begin{aligned}\mathbf{R}_{\mathbf{y}-\boldsymbol{\lambda}} =& |\alpha_0|^2[\mathbf{s}\odot\mathbf{p}(\nu)][\mathbf{s}\odot\mathbf{p}(\nu)]^H \\ &+\boldsymbol{\lambda}\boldsymbol{\lambda}^H + \mathbf{R} - 2\Re\left(\alpha_0[\mathbf{s}\odot\mathbf{p}(\nu)]\boldsymbol{\lambda}^H\right).\end{aligned} \tag{14}$$

As a result, in order to recover the backscattering coefficient α_0 and the normalized Doppler shift ν, one can minimize the following non-convex criterion with respect to α_0 and ν

$$\left\|\bar{\mathbf{R}}_{\mathbf{y}-\boldsymbol{\lambda}} - \mathbf{F}(\alpha_0,\nu)\right\|_F \tag{15}$$

where

$$\begin{aligned}\mathbf{F}(\alpha_0,\nu) = \mathcal{N}\Bigg(&|\alpha_0|^2[\mathbf{s}\odot\mathbf{p}(\nu)][\mathbf{s}\odot\mathbf{p}(\nu)]^H \\ &+\boldsymbol{\lambda}\boldsymbol{\lambda}^H + \mathbf{R} - 2\Re\left(\alpha_0[\mathbf{s}\odot\mathbf{p}(\nu)]\boldsymbol{\lambda}^H\right)\Bigg).\end{aligned}$$

IV. THE PROPOSED ALGORITHM

In this section, we discuss our proposed algorithm to jointly recover the received signal $\mathbf{y}$ and the radar parameters of the target in the desired range-azimuth bin, namely the backscattering coefficient α_0 and normalized Doppler shift ν.

In order to find the optimal IV filter $\mathbf{w}$ [24], [25], one can minimize the MSE in (10) with respect to $\mathbf{w}$ to find

$$\mathbf{w} = \mathbf{R}^{-1}(\mathbf{s}\odot\mathbf{p}(\nu)). \tag{16}$$

In spite of the MMF approach to estimating the backscattering coefficient in (9), as it was discussed earlier, due to one-bit sampling, there is no direct access to the unquantized received signal $\mathbf{y}$. Hence, our proposed algorithm shall rely only on the one-bit sampled data $\boldsymbol{\gamma}$ and the threshold vector $\boldsymbol{\lambda}$ to recover the received signal and radar parameters. Thus, in pursuance of estimating the radar parameters, we define the following weighted-least-squares objective function

$$\begin{aligned}Q(\mathbf{y},&\alpha_0,\nu) = \\ &[\mathbf{y}-\alpha_0(\mathbf{s}\odot\mathbf{p}(\nu))]^H\mathbf{R}^{-1}[\mathbf{y}-\alpha_0(\mathbf{s}\odot\mathbf{p}(\nu))].\end{aligned} \tag{17}$$

The above objective function has the following properties:

1) It does not require any knowledge of the un-quantized received signal $\mathbf{y}$.
2) It is a function of the received signal $\mathbf{y}$, the backscattering coefficient α_0, and the normalized Doppler shift ν. This facilitates their joint recovery.
3) For given $\mathbf{y}$ and ν, it can be verified that the optimal α_0 is given exactly by the MMF estimate of α_0, which is basically (9) with substitution of (16).
4) It is in agreement with (and enforces the) system model in (1). As for the unwanted interference in (1), one can rewrite them as

$$\mathbf{y} - \alpha_0[\mathbf{s}\odot\mathbf{p}(\nu)] = \mathbf{c} + \mathbf{n}. \tag{18}$$

The weighted-least-squares objective of (17) penalizes the model mismatch based on the second-order statistics of the interference, derived as

$$\begin{aligned}&\mathbb{E}\left\{(\mathbf{c}+\mathbf{n})(\mathbf{c}+\mathbf{n})^H\right\} \\ &=\mathbb{E}\left\{\mathbf{c}\mathbf{c}^H\right\} + \mathbb{E}\left\{\mathbf{n}\mathbf{n}^H\right\} = \mathbf{R}\end{aligned} \tag{19}$$

where $\mathbf{R}$ is the same as in (7).

Therefore, the problem of jointly estimating the received signal and the moving target radar parameters boils down to the optimization problem:

$$\begin{aligned}\min_{\alpha_0,\mathbf{y},\nu} \quad & [\mathbf{y}-\alpha_0(\mathbf{s}\odot\mathbf{p}(\nu))]^H\mathbf{R}^{-1}[\mathbf{y}-\alpha_0(\mathbf{s}\odot\mathbf{p}(\nu))] \\ \text{s.t.} \quad & \boldsymbol{\Omega}_r(\mathbf{y}_r - \boldsymbol{\lambda}_r) \geq \mathbf{0}, \\ & \boldsymbol{\Omega}_i(\mathbf{y}_i - \boldsymbol{\lambda}_i) \geq \mathbf{0},\end{aligned} \tag{20}$$

where $(\mathbf{y}_r, \mathbf{y}_i)$ and $(\boldsymbol{\lambda}_r, \boldsymbol{\lambda}_i)$ are the real and imaginary parts of $\mathbf{y}$ and $\boldsymbol{\lambda}$, respectively. Also, $\boldsymbol{\Omega}_r = \mathbf{Diag}(\boldsymbol{\gamma}_r)$ and $\boldsymbol{\Omega}_i = \mathbf{Diag}(\boldsymbol{\gamma}_i)$, where $\mathbf{Diag}(\cdot)$ denotes the diagonalization operator that produces a diagonal matrix as output which has only diagonal entries same as in its input vector.

To solve the optimization problem in (20), we will utilize a cyclic optimization over $\mathbf{w}$, $\mathbf{y}$, and ν, until convergence. By applying the substitution of the optimal α_0 in (9) into the objective function of (17), we obtain a simplified objective function, viz.

$$Q(\mathbf{w},\mathbf{y},\nu) = \left\|\mathbf{R}^{-1/2}\left(\mathbf{I} - \frac{[\mathbf{s}\odot\mathbf{p}(\nu)]\mathbf{w}^H}{\mathbf{w}^H[\mathbf{s}\odot\mathbf{p}(\nu)]}\right)\mathbf{y}\right\|_2^2. \tag{21}$$

Note that for given $\mathbf{y}$ and ν, the optimal IV filter $\mathbf{w}$ is given by (16). On the other hand, for given $\mathbf{w}$ and ν, the minimization problem for estimating $\mathbf{y}$ becomes

$$\begin{aligned}\min_{\mathbf{y}} \quad & \left\|\mathbf{R}^{-1/2}\left(\mathbf{I} - \frac{[\mathbf{s}\odot\mathbf{p}(\nu)]\mathbf{w}^H}{\mathbf{w}^H[\mathbf{s}\odot\mathbf{p}(\nu)]}\right)\mathbf{y}\right\|_2^2 \\ \text{s.t.} \quad & \boldsymbol{\Omega}_r(\mathbf{y}_r - \boldsymbol{\lambda}_r) \geq \mathbf{0}, \\ & \boldsymbol{\Omega}_i(\mathbf{y}_i - \boldsymbol{\lambda}_i) \geq \mathbf{0}.\end{aligned} \tag{22}$$

It can easily be verified that the above minimization problem is a convex linearly-constraint quadratic program with respect to the variable $\mathbf{y}$, which can be solved efficiently using numerous methods available in the literature; e.g. see [26].

Next, in order to recover the normalized Doppler shift ν for given $\mathbf{w}$ and $\mathbf{y}$, one can rewrite the minimization problem of (20) on ν as

$$
\begin{aligned}
\min_{\nu} \quad & g(\nu) \\
\text{s.t.} \quad & \mathbf{p}(\nu) = \begin{bmatrix} e^{j2\pi(0)\nu} & e^{j2\pi(1)\nu} & \dots & e^{j2\pi(N-1)\nu} \end{bmatrix}^T
\end{aligned} \tag{23}
$$

where

$$
g(\nu) \triangleq \begin{bmatrix} 1 \\ \mathbf{p}(\nu) \end{bmatrix}^H \begin{bmatrix} 0 & -(\hat{\alpha}_0\mathbf{s})^T \odot (\mathbf{y}^H\mathbf{R}^{-1}) \\ -(\hat{\alpha}_0\mathbf{s})^* \odot (\mathbf{R}^{-1}\mathbf{y}) & |\hat{\alpha}_0|^2\mathbf{R}^{-1} \odot (\mathbf{s}\mathbf{s}^H)^* \end{bmatrix} \begin{bmatrix} 1 \\ \mathbf{p}(\nu) \end{bmatrix}
$$

and $\hat{\alpha}_0$ is calculated using (9). The above optimization problem on ν resembles the estimation of direction-of-arrival (DOA) in uniform linear arrays. Thus, it can be tackled using any of the multitudinous methods derived for estimating DOAs; e.g. see [27]. Note that the cyclic optimization iterations with respect to $\mathbf{w}$, $\mathbf{y}$, and ν should be continued until a convergence or another stop criteria is satisfied. Once the optimal $\mathbf{w}$, $\mathbf{y}$, and ν are found, one can estimate the backscattering coefficient α_0 using (9).

Finally, we need to have the threshold vector $\boldsymbol{\lambda}$ designed in such a way that it is the most informative about the received signal. From an information-theoretic perspective, we want the probability of observing both $+1$ and -1 at the output of the sampler to be as close as possible. For this to happen, the optimal choice of $\boldsymbol{\lambda}$ should divide the signal space into subspaces with similar cardinalities. Such a decision can be difficult when the dimension of the signal, N, grows large. However, a reasonable approximation of the optimal $\boldsymbol{\lambda}$ can be obtained when it is assumed to be a random variable that follows the statistics and demeanors of the received signal $\mathbf{y}$. Additionally, the received signal $\mathbf{y}$ can be considered a to be a Gaussian random variable. Therefore, we generate the threshold vector $\boldsymbol{\lambda}$ from a Gaussian distribution with the following statistics:

$$
\begin{aligned}
\mathbb{E}\{\boldsymbol{\lambda}\} &= \mathbb{E}\{\mathbf{y}\} = \mathbb{E}\{\alpha_0\}(\mathbf{s} \odot \mathbb{E}\{\mathbf{p}(\nu)\}), \\
\mathrm{Cov}(\boldsymbol{\lambda}) &= \mathrm{Cov}(\mathbf{y}) \\
&= \mathbb{E}\{|\alpha_0|^2\}\left[(\mathbf{s}\mathbf{s}^H) \odot \left(\mathbb{E}\{\mathbf{p}(\nu)\mathbf{p}^H(\nu)\}\right)\right] \\
&\quad +\mathbf{R}.
\end{aligned} \tag{24}
$$

That is, the design of $\boldsymbol{\lambda}$ is governed by the *expected value* of the radar parameters, which can be very useful, especially in tracking applications.

The proposed algorithm is summarized in Algorithm 1 for reader's convenience.

Algorithm 1

One-Bit Moving Target Radar Parameter Estimation

Step 0: Generate the threshold vector $\boldsymbol{\lambda}$ randomly based on the statistics in (24).
Step 1: For fixed $\mathbf{y}$ and ν, compute the optimal IV filter $\mathbf{w}$ using (16).
Step 2: For fixed $\mathbf{w}$ and ν, compute the optimal vector $\mathbf{y}$ by solving the minimization problem of (22) over the variable $\mathbf{y}$.
Step 3: For fixed $\mathbf{w}$ and $\mathbf{y}$, compute the optimal target normalized Doppler shift ν by minimizing the criterion in (23).
Step 4: If convergence is reached, goto Step 5, otherwise goto Step 1.
Step 5: For fixed $\mathbf{w}$, $\mathbf{y}$, and ν, find the optimal target backscattering coefficient α_0 using (9).
Step 6: In case of tracking, set $\boldsymbol{\lambda}$ according to (24) and goto Step 1.

V. Numerical Results

In this section, we study the performance of the proposed algorithm of Section IV. In order to do this, we compare the errors associated with estimations obtained by 1) the proposed algorithm, 2) the Bussgang-theorem-aided method of Section III, referred to as Bussgang-aided, and 3) using the unquantized received signal $\mathbf{y}$, which is denoted by $\infty - precision$.

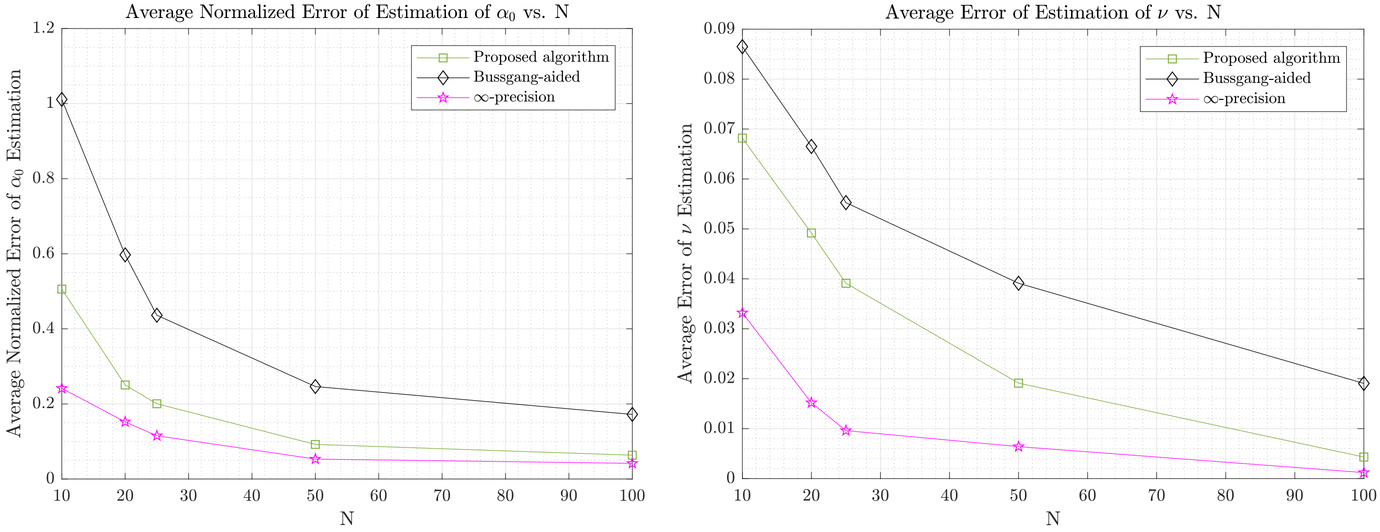
For simulations, we assume that the signal-independent interference term is a white Gaussian noise with a variance of .1, and that the transmit sequence $\mathbf{s}$ is generated using the method in [28]. Moreover, the number of the interfering rings N_c and azimuth sectors L are assumed to be 2 and 10, respectively. A uniformly distributed clutter is assumed with $\sigma^2_{(k,l)} = 100$ for all (k,l) pairs and their normalized Doppler shifts are assumed to have a uniform distribution over the interval $\Omega_c = [-.1.1]$; for further details see [29]. Also, to comply with the Monte-Carlo test requirements, the result of each of the algorithms is averaged over 100 runs.

Figure 1(a) shows the normalized estimation error of the backscattering coefficient α_0 defined by $|\alpha_0 - \hat{\alpha}_0|/|\alpha_0|$, where $\hat{\alpha}_0$ denotes the estimate of α_0. Similarly, Figure 1(b) shows the same metric in estimation of the normalized Doppler shift ν. The plots in Figure 2 show the results of estimating the radar parameters using the mentioned methods in a Monte-Carlo trial along with the true value of the parameter for $N \in \{50, 100, 1000\}$. The upper plots of Figure 2 shows the estimation of α_0 on a complex plane while the lower plots show the estimation of $e^{j2\pi\nu}$, where the estimation results of different methods are shown with different radii for aesthetic purposes.

It can be verified from the figures that the error of estimation of α_0 and ν decreases as N grows larger. More interestingly, as N increases, the estimation performance of our method reaches that of the $\infty - precision$ case where we have the unquantized received signal at hand. This behavior is expected because as N grows large, more information will be available to be gleaned from the one-bit sampled data about the received signal, which facilitates a more accurate recovery.

VI. Conclusion

Many modern applications require very fast sampling of data. However, the cost and energy consumption of the current ADCs grows exponentially with the sampling resolution. To address this issue, there is a growing interest in resorting to one-bit sampling of data that uses an inexpensive and energy-efficient comparator in lieu of complex ADC structures. One such application, namely moving target radar parameter estimation, was studied in this paper. An algorithm to estimate the target radar parameters was developed. Several numerical results were provided to verify the favorable performance of the proposed algorithm, especially when the length of transmit sequence grows large. Thus, it was shown that one-bit sampling can be used efficiently as a replacement for the conventional ADCs.



(a) (b)

Fig. 1: Performance comparison of moving target parameter estimation using the Bussgang-aided method, the proposed algorithm, and in the $\infty - precision$ case: (a) average normalized estimation error ($|\alpha_0 - \hat{\alpha}_0|/|\alpha_0|$) of backscattering coefficient α_0, (b) average estimation error of the normalized Doppler shift ν.

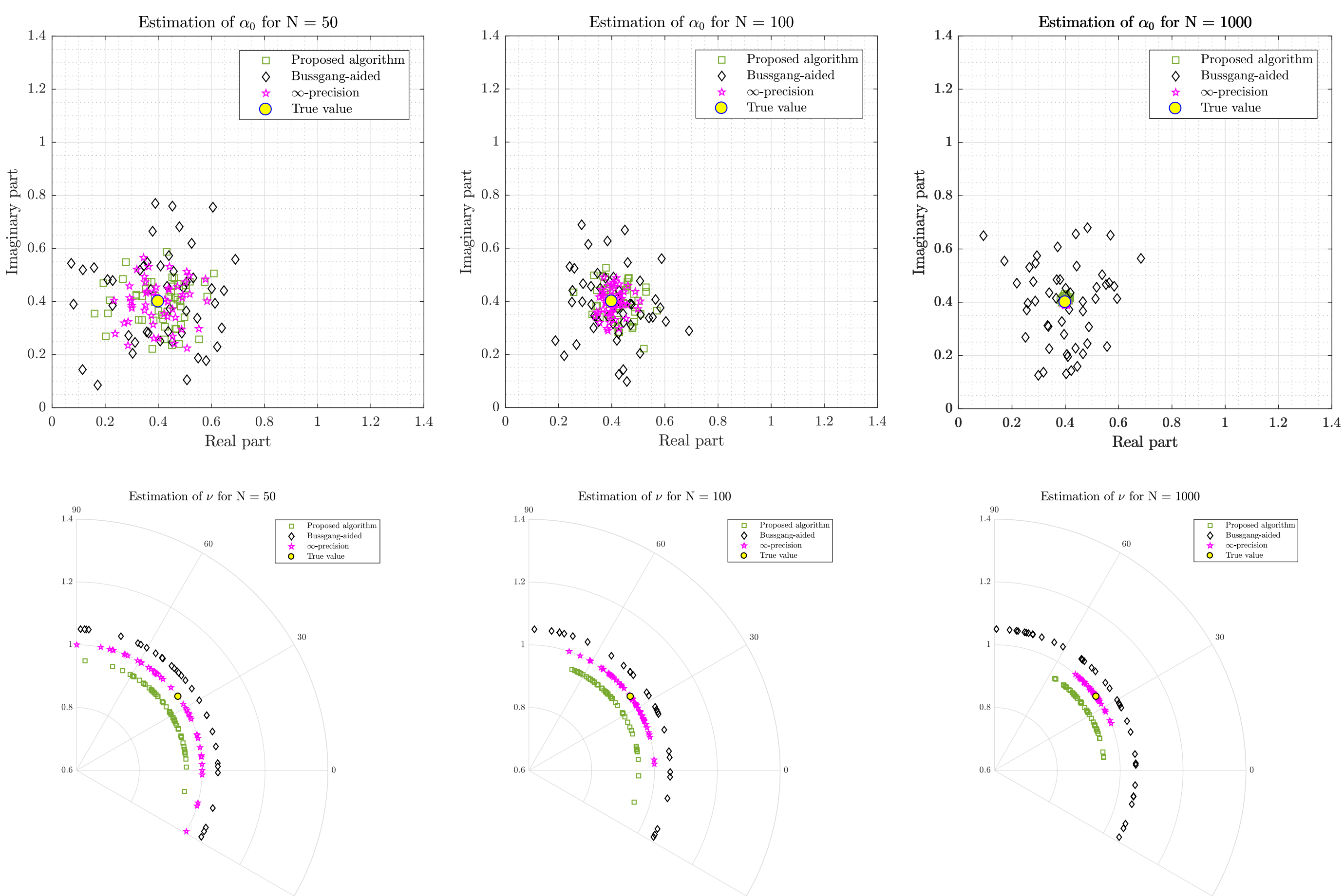


Fig. 2: Performance comparison of moving target parameter estimation using the Bussgang-aided method, the proposed algorithm, and in the $\infty - precision$ case for $N \in \{50, 100, 1000\}$. The upper plots show the estimation results for the backscattering coefficient α_0 on the complex plane while the lower plots show the target Doppler shift estimates on the unit circle $e^{j2\pi\nu}$ and polar plane. For visual clarity, different radii are used for different approaches.

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